CMPT 409/981: Quantum Circuits and Compilation

Lecture 1

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What is this course about?

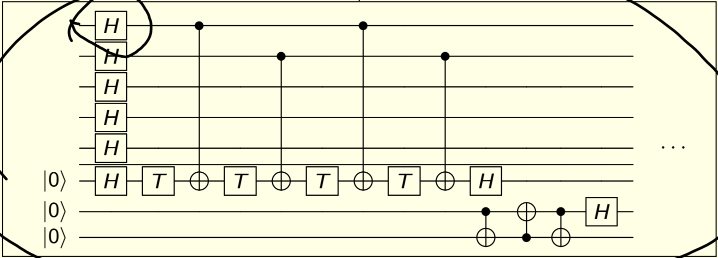
Algorithm

GROVER(f, n)

1. Prepare state $|\psi\rangle = \frac{1}{2^n} \sum_{x \in \mathbb{F}_2^n} |x\rangle$
2. For i in $[0, r(2^n)]$ do
   2.1 Apply $U_f : |x\rangle \mapsto (-1)^{f(x)}|x\rangle$

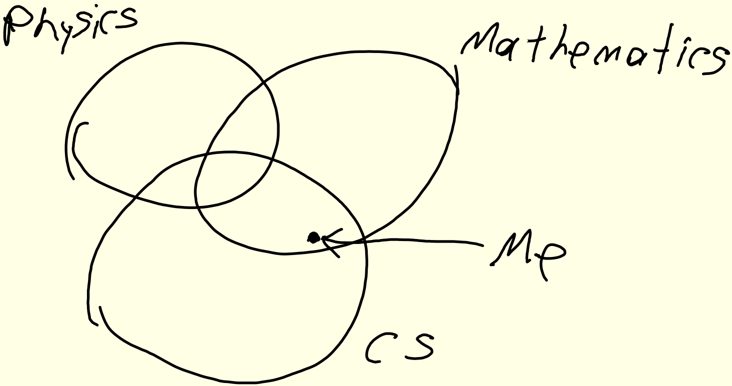
$\vdots$

Implementation



Who am I?

I work on tools for formally reasoning about and modeling quantum computations



Is this course for me?

Probably yes, if:

- You have a good handle on linear algebra
- Abstract mathematics and theoretical CS doesn't scare you
  - E.g. $\mathbb{Q}[\sqrt{2}]$ ⟵ $a + b\sqrt{2}$
  - E.g. $O(n \log n)$
- You're comfortable reading research papers

How will this course work?

- Mixture of lectures (approx. 1/3) to introduce topics and background material
- Rest of the course will be readings + presentations + discussion

Course website: https://www.cs.sfu.ca/~meamy/f22/cmpt981/

- Constitutes the syllabus
- Check regularly!

How am I being graded?

- 40% final project
- 20% class participation
  - For discussion portion
- 35% exercises
  - mix of problems and paper summaries/presentations
- **5% collaborative class notes**
  - Experiment!

Quantum computation

What is computation

What is computation (classical or otherwise)?

- A **physical** process of calculation

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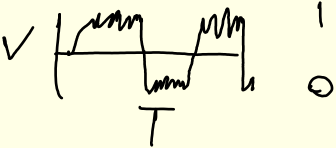
  - E.g., 0 for low voltage, 1 for high voltage



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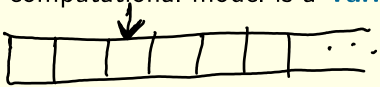
- A standard computational model is a **Turing machine**



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Conjecture (The strong Church–Turing thesis)

A probabilistic Turing machine can efficiently simulate any realistic model of computation.

The birth of quantum computers

(Feynman, 1982)

> *a quantum process is a physical process of calculation which cannot be simulated efficiently by any classical model of computation*

violation of SCTT

The birth of quantum computers

(Feynman, 1982)

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Next came **practical** calculations using **quantum systems**

- (Shor, 1994) integer factorization
- (Lloyd, 1996) Universal quantum simulation
- (Grover, 1996) Unstructured search algorithm
- Computing knot invariants, linear systems, etc...

Quantum computation

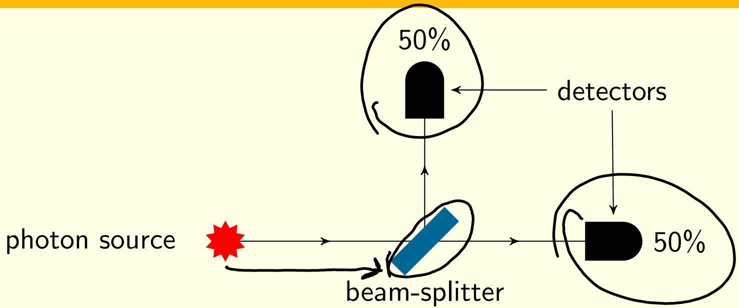
Quantum computers can (in principle) be built out of any physical system with at least **2** identifiable and measurable states, e.g.,

- electron spin (up or down)
- light polarization (horizontal or vertical)

We typically call these states $|0\rangle$ and $|1\rangle$

Unlike classical bits, a quantum bit (**qubit**) can be in states $|0\rangle$, $|1\rangle$ or a **superposition** of both

Example: photons



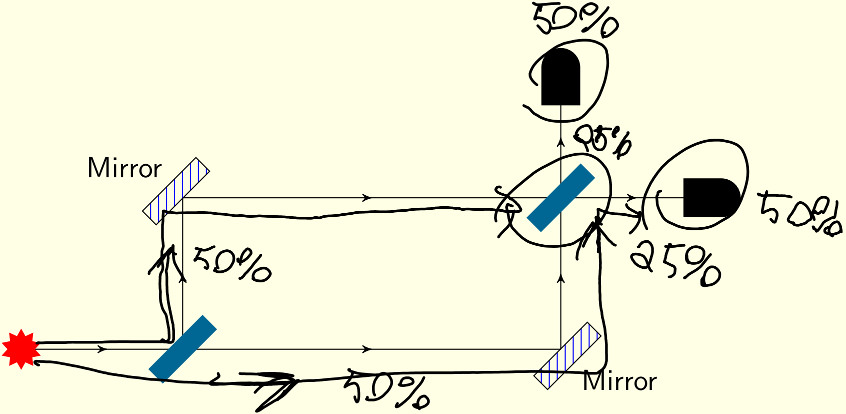
A system with two states can be built by placing photon detectors at two locations

A single photon sent into a beam-splitter will either

- continue straight through, or
- be reflected

with **equal probability**

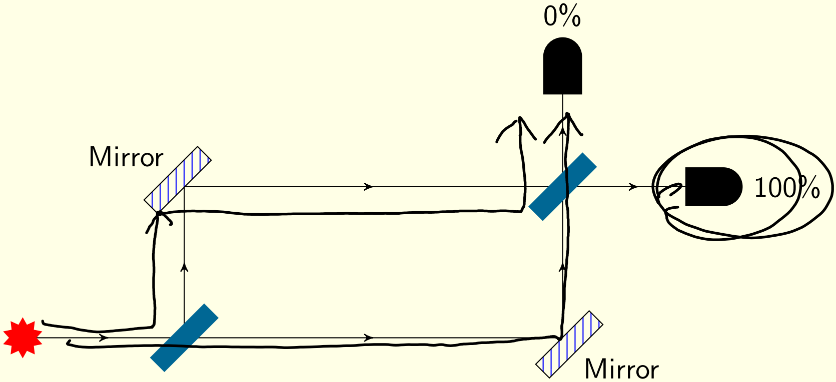
Interference



Where will a single photon be detected?

- Classical intuition says equal probability at either location

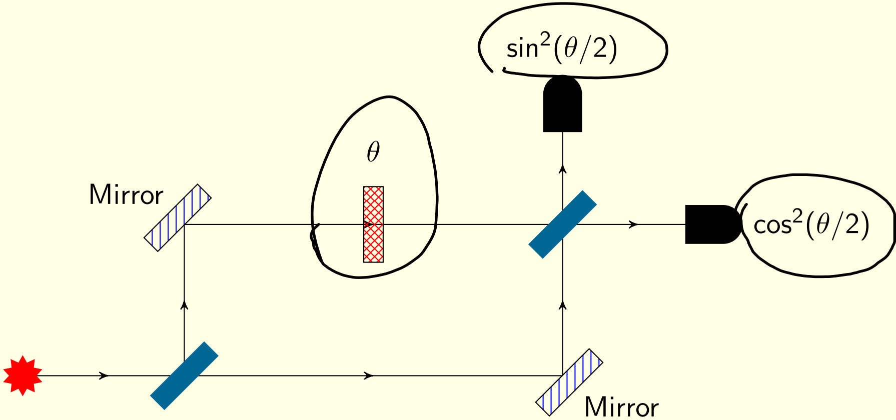
Interference



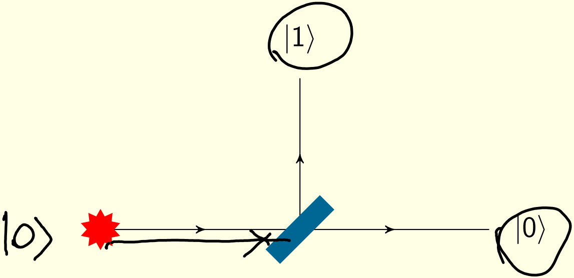
Experimentally (depending on the material and photon source), it will always appear at the lower detector

- Intuition is that the photon took **both paths simultaneously**
- **Interference** causes paths to the upper detector to cancel

Interference



Adding a **phase shift** along one path changes the results

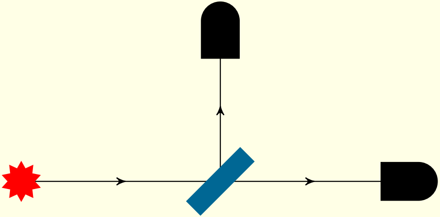


The beam-splitter sends a photon along a **superposition** of paths

- let $|0\rangle$ denote the transmitted path
- let $|1\rangle$ denote the reflected path
- we can model the beam-splitter mathematically as mapping

$$|0\rangle \mapsto \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle \qquad |1\rangle \mapsto \frac{i}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

Measurement



When we **measure** a photon in a superposition of paths

$$\alpha|0\rangle + \beta|1\rangle, \qquad \alpha, \beta \in \mathbb{C}$$

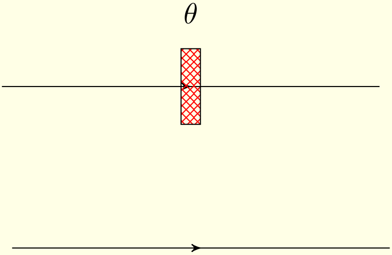
we detect the photon in the

- $|0\rangle$ path with probability $|\alpha|^2$
- $|1\rangle$ path with probability $|\beta|^2$

α and β are called **probability amplitudes** and their measurement outcomes form a **probability distribution**,

$$|\alpha|^2 + |\beta|^2 = 1$$

Phase shift



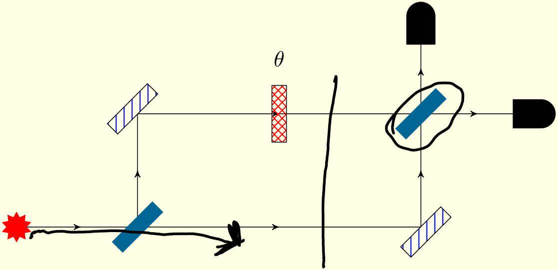
- The **phase shift** on the $|1\rangle$ path rotates the polarization (the phase) by an angle of θ

$$|0\rangle \mapsto |0\rangle$$
$$|1\rangle \mapsto e^{i\theta}|1\rangle$$

- Operations are **linear**, e.g.,

$$\alpha|0\rangle + \beta|1\rangle \mapsto \alpha|0\rangle + \beta e^{i\theta}|1\rangle$$

Putting it all together



After the phase shift, the state is

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{ie^{i\theta}}{\sqrt{2}}|1\rangle$$

The second beam-splitter then acts **linearly**, acting independently on photons ~~in either~~ path:

$$|0\rangle \mapsto \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle \qquad |1\rangle \mapsto \frac{i}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

Interference

Final state when the detector is reached:

$$\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle\right) + \frac{ie^{i\theta}}{\sqrt{2}}\left(\frac{i}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)$$

$$= \frac{1}{2}|0\rangle + \frac{i}{2}|1\rangle + \frac{-e^{i\theta}}{2}|0\rangle + \frac{ie^{i\theta}}{2}|1\rangle$$

$$= \frac{1-e^{i\theta}}{2}|0\rangle + \frac{i+ie^{i\theta}}{2}|1\rangle$$

Interference

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$$= \frac{1}{1}|0\rangle + \frac{i}{2}|1\rangle + \frac{-e^{i\theta}}{2}|0\rangle + \frac{ie^{i\theta}}{2}|1\rangle$$
$$= \frac{1-e^{i\theta}}{2}|0\rangle + \frac{i+ie^{i\theta}}{2}|1\rangle$$

$$\cancel{\frac{1-1}{2}}|0\rangle + \frac{i+i}{2}|1\rangle$$

If $\theta = 0$, then $e^{i\theta} = 1$ and the $|0\rangle$ **paths destructively interfere**, leaving only state $|1\rangle$

Interference

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In general, a photon is detected in the

- $|0\rangle$ path with probability $|\frac{1-e^{i\theta}}{2}|^2 = \sin^2(\theta/2)$
- $|1\rangle$ path with probability $|\frac{i+ie^{i\theta}}{2}|^2 = \cos^2(\theta/2)$

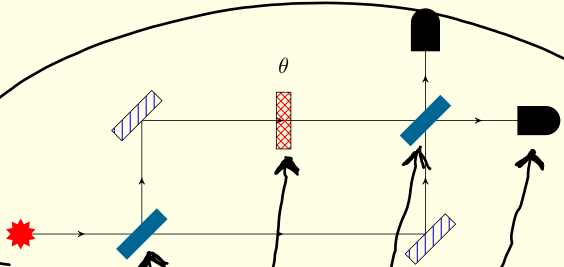
Quantum parallelism and algorithms

Quantum algorithms generally work by...

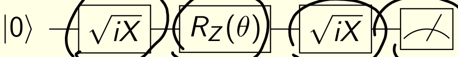
- Preparing some superposition of **classical** states
- Running some computation (by linearity) on **all states at once**
- Using **interference** to extract the desired result

These algorithms are described as high-level **circuits** or via **circuit description languages**

As a quantum circuit



We write this as a sequence of **gates** in the circuit model:

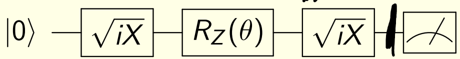


where

$$\sqrt{iX} = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix} \qquad R_Z(\theta) = \begin{bmatrix}1 & 0\\ 0 & e^{i\theta}\end{bmatrix}$$

Calculation by linear algebra

We can evaluate the circuit



up to the final **measurement** by matrix multiplication as below:

$$\frac{1}{2}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & e^{i\theta}\end{bmatrix}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & e^{i\theta}\end{bmatrix}\begin{bmatrix}1\\ i\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix}\begin{bmatrix}1\\ ie^{i\theta}\end{bmatrix}=\frac{1}{2}\begin{bmatrix}1-e^{i\theta}\\ i+ie^{i\theta}\end{bmatrix}$$

Final note

Have a paper on the topic of circuits & compilation you would like to read in this course? Let me know!